

参考答案

1、ABCD A CCBAD DD

13、 $a \in (2, 6)$

14、 $1/3$

15、 $a_n = \begin{cases} 2, n=1, \\ 4n-1, n \geq 2. \end{cases}$

16、 $\frac{\sqrt{2}}{10}$

17、(1) $-\frac{4\sqrt{2}}{9}$.

(2) $\frac{4+6\sqrt{2}}{15}$.

(I) $\because \alpha \in (\frac{\pi}{2}, \pi)$, 且 $\sin \alpha = \frac{1}{3}$, $\therefore \cos \alpha = -\frac{2\sqrt{2}}{3}$, -----2 分

于是 $\sin 2\alpha = 2\sin \alpha \cos \alpha = -\frac{4\sqrt{2}}{9}$;

(II) $\because \alpha \in (\frac{\pi}{2}, \pi), \beta \in (0, \frac{\pi}{2})$, $\therefore \alpha + \beta \in (\frac{\pi}{2}, \frac{3}{2}\pi)$, 结合 $\sin(\alpha + \beta) = -\frac{3}{5}$ 得: $\cos(\alpha + \beta) = -\frac{4}{5}$,

于是

$$\sin \beta = \sin[(\alpha + \beta) - \alpha] = \sin(\alpha + \beta)\cos \alpha - \cos(\alpha + \beta)\sin \alpha = -\frac{3}{5} \cdot (-\frac{2\sqrt{2}}{3}) - (-\frac{4}{5}) \cdot \frac{1}{3} = \frac{4+6\sqrt{2}}{15}.$$

18、解: (1) 设 $\{a_n\}$ 的公差为 d ,

$$\text{由已知条件, 得 } \begin{cases} a_1 + d = 1, \\ a_1 + 4d = -5, \end{cases}$$

解得 $a_1 = 3, d = -2$.

所以 $a_n = a_1 + (n-1)d = -2n + 5$.

$$(2) S_n = na_1 + \frac{n(n-1)}{2}d = -n^2 + 4n = 4 - (n-2)^2.$$

所以 $n=2$ 时, S_n 最大, 且最大值为 4.

19、(1) $AE = 1$ (2) $\frac{\sqrt{6}}{6}$

解: (1) 由 $2(b^2 + c^2) = 2a^2 + \sqrt{6}bc$, 可知 $\cos A = \frac{\sqrt{6}}{4}$

从而 $\sin A = \frac{\sqrt{10}}{4}$

$$\text{由 } S_{\triangle ACE} = \frac{1}{2} \times AE \times \sqrt{6} \times \frac{\sqrt{10}}{4} = \frac{\sqrt{15}}{4}, \therefore AE = 1$$

(2) $\because BE = BC, \therefore \angle BCE = \angle BEC,$

$$\therefore \sin \angle BCE = \sin \angle BEC = \sin \angle AEC,$$

$$\therefore \frac{\sin \angle ACE}{\sin \angle BCE} = \frac{\sin \angle ACE}{\sin \angle AEC} = \frac{AE}{AC} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}.$$

20、(I) $\frac{\pi}{3}$; (II) 6.

(I) 由 正弦定理得 $2\cos B(\sin A \cos C + \sin C \cos A) = \sin B,$

所以 $2\cos B \sin B = \sin B,$ 因 $\sin B > 0,$ 故 $\cos B = \frac{1}{2}.$

又 $B \in (0, \pi),$ 故 $B = \frac{\pi}{3}.$

(II) $b = 2,$ 由正弦定理 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$ 及 $A = \alpha, C = \pi - A - B = \frac{2\pi}{3} - \alpha$ 得 $\frac{a}{\sin \alpha} =$

$$\frac{b}{\sin \frac{\pi}{3}} = \frac{c}{\sin(\frac{2\pi}{3} - \alpha)}, \therefore a = \frac{4}{\sqrt{3}} \sin \alpha, c = \frac{4}{\sqrt{3}} \sin(\frac{2\pi}{3} - \alpha),$$

$$\therefore \triangle ABC \text{ 周长 } l = f(\alpha) = a + b + c = \frac{4}{\sqrt{3}} \sin \alpha + 2 + \frac{4}{\sqrt{3}} \sin(\frac{2\pi}{3} - \alpha)$$

$$= \frac{4}{\sqrt{3}} (\sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha) + 2 = \frac{4}{\sqrt{3}} (\frac{3}{2} \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha) + 2$$

$$= \frac{4}{\sqrt{3}} \sqrt{3} (\frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha) + 2 = 4 \sin(\alpha + \frac{\pi}{6}) + 2$$

$$\because 0 < \alpha < \frac{2\pi}{3} \therefore \text{当 } \alpha + \frac{\pi}{6} = \frac{\pi}{2} \text{ 即 } \alpha = \frac{\pi}{3} \text{ 时 } l_{\max} = f(\frac{\pi}{3}) = 4 + 2 = 6$$

所以 $\triangle ABC$ 周长 $l = f(\alpha)$ 的最大值为 6.

21.[解] (1)证明: 由题设知 $a_n a_{n+1} = \lambda S_n - 1, a_{n+1} a_{n+2} = \lambda S_{n+1} - 1,$ 2 分

两式相减得 $a_{n+1}(a_{n+2} - a_n) = \lambda a_{n+1},$

由于 $a_{n+1} \neq 0,$ 所以 $a_{n+2} - a_n = \lambda.$ 5 分

(2)由题设知 $a_1 = 1, a_1 a_2 = \lambda S_1 - 1,$

可得 $a_2 = \lambda - 1.$

由(1)知, $a_3 = \lambda + 1.$ 7 分

令 $2a_2 = a_1 + a_3,$ 解得 $\lambda = 4.$

故 $a_{n+2} - a_n = 4,$ 由此可得 $\{a_{2n-1}\}$ 是首项为 1, 公差为 4 的等差数列, $a_{2n-1} = 4n - 3;$ 9 分

$\{a_{2n}\}$ 是首项为 3, 公差为 4 的等差数列, $a_{2n} = 4n - 1.$

所以 $a_n = 2n - 1, a_{n+1} - a_n = 2,$

因此存在 $\lambda = 4,$ 使得数列 $\{a_n\}$ 为等差数列. 12 分

$$22. \text{解: (1) } \because \triangle AMN \cong \triangle A'MN, \therefore \angle AMN = \angle A'MN = \frac{\pi}{3},$$

$$\therefore \angle BMA' = \frac{\pi}{3}, \therefore BM = \frac{1}{2}A'M = \frac{1}{2}AM.$$

$$\therefore AM = \frac{2}{3}AB = \frac{2}{3}a,$$

$$\because AB = a, BC = \sqrt{3}a, \angle B = \frac{\pi}{2}, \therefore \angle A = \frac{\pi}{3},$$

$\therefore \triangle AMN$ 是等边三角形,

$$\therefore S = 2S_{\triangle AMN} = 2 \times \frac{\sqrt{3}}{4} \times \frac{4a^2}{9} = \frac{2\sqrt{3}a^2}{9}.$$

$$(2) \because \angle BMA' = \pi - 2\theta, AM = A'M,$$

$$\therefore BM = A'M \cos \angle BMA' = -AM \cos 2\theta.$$

$$\because AM + BM = a, \text{ 即 } AM(1 - \cos 2\theta) = a,$$

$$\therefore AM = \frac{a}{1 - \cos 2\theta} = \frac{a}{2\sin^2 \theta}.$$

$$\text{在 } \triangle AMN \text{ 中, 由正弦定理可得: } \frac{AN}{\sin \theta} = \frac{AM}{\sin(\pi - \frac{\pi}{3} - \theta)},$$

$$\therefore AN = \frac{AM \sin \theta}{\sin(\frac{2\pi}{3} - \theta)} = \frac{a}{2\sin \theta \sin(\frac{2\pi}{3} - \theta)},$$

$$\begin{aligned} \text{令 } f(\theta) &= 2\sin \theta \sin(\frac{2\pi}{3} - \theta) = 2\sin \theta (\frac{\sqrt{3}}{2}\cos \theta + \frac{1}{2}\sin \theta) = \frac{\sqrt{3}}{2}\sin 2\theta + \frac{1 - \cos 2\theta}{2} \\ &= \sin(2\theta - \frac{\pi}{6}) + \frac{1}{2}. \end{aligned}$$

$$\because \theta \in (\frac{\pi}{4}, \frac{\pi}{2}), \therefore \text{当 } 2\theta - \frac{\pi}{6} = \frac{\pi}{2} \text{ 即 } \theta = \frac{\pi}{3} \text{ 时 } f(\theta) \text{ 取最大值,}$$

$$\therefore \text{当 } \theta = \frac{\pi}{3} \text{ 时 } AN \text{ 最短, 此时 } \triangle AMN \text{ 是等边三角形, } MN = AM = \frac{2}{3}a.$$